Infinitely many Zhang functions resulting in various ZNN models for time-varying matrix inversion with link to Drazin inverse

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**Abstract**

In this Letter, by generalizing the notion of Zhang functions (ZFs) from previous work, a novel general-form Zhang function (NGFZF) is proposed, developed and investigated. Specifically, based on the NGFZF, infinitely many ZFs (as error functions) can be readily generated by successively selecting the different values of its parameters. Besides, by employing the NGFZF, a novel general-form Zhang neural net (NGFZNN) is proposed and studied for real-time solution of a time-varying matrix inverse (also termed, Zhang matrix inverse, ZMI). Moreover, a link between ZMI and Drazin inverse is discovered and further generalized to solve for the time-varying Drazin inverse (TVDI).

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1. Introduction

Lately, the recurrent neural net approach has shown a huge potential for solving online a variety of computational problems owing to its parallel distributed nature and convenience of hardware implementation [1–3]. As a result, a special class of recurrent neural net, termed Zhang neural net (ZNN), has been formally proposed for solving various time-varying problems [4]. Recently, as for the time-varying problem solving that is related to division (or, generalized division), where the divisor (or, generalized divisor) may vary and pass through zero, the notion of Zhang problem solving (ZPS) has been formally proposed. For instance, time-varying inverse square root finding [5], time-varying reciprocal computation [6], and time-varying matrix inversion [7], all can be regarded as specific topics of ZPS. By following the ZPS notion, the time-varying matrix inverse associated with ZNN is often termed Zhang matrix inverse (ZMI) for comparison purposes. Note that ZMI is generally regarded as an essential part for various schemes encountered in science and engineering fields, such as time-varying pseudoinverse based schemes for robotic redundancy resolution [8].

Facing the ZPS directly, we consider the real-time solution of ZMI problem in this Letter, which is formulated as

$$A(t)X(t) - I = 0,$$

where $A(t) \in \mathbb{R}^{n \times n}$ denotes a smoothly time-varying coefficient matrix and $I \in \mathbb{R}^{n \times n}$ denotes the identity matrix. In